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by

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Need vs. Merit: The Large Core of College Admissions Markets*

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Abstract

We study college admissions markets, where colleges offer multiple funding levels. Colleges wish to recruit the best-qualified students subject to budget and capacity constraints. Student-proposing deferred acceptance is stable and strategy-proof for students, but the set of stable allocations is large and the scope for manipulation by colleges is substantial, even in large markets. Under deferred acceptance, truthful colleges allocate funding based on merit. Successful manipulations consider applicants' outside options (specifically need) when allocating funding. In Hungary, where the centralized clearinghouse uses deferred acceptance, choosing another stable allocation would increase the number of admitted students by at least 3%.

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1 Introduction

In recent years, a growing number of students are being assigned to schools through centralized clearinghouses. The success of such clearinghouses crucially relies on the use of a stable matching mechanism.[1] Stability is also useful for predicting behavior in decentralized matching markets.[2] Empirical and theoretical studies suggest that all applicants, save a handful, receive the same assignment in all stable allocations.[3] This finding, that the set of stable allocations is small, has several implications. First, a designer who wishes to implement a stable outcome has limited scope for further design. Second, agents have little incentive to collect information on others. A closely related result is that incentives to misreport one's preferences to the student-proposing deferred acceptance mechanism (henceforth DA[4]) are minimal.[5]

The above-mentioned results apply to settings where agents on both sides of a two-sided matching market (e.g., students and schools, men and women, etc.) have preferences over potential partners from the other side. However, the environments studied and designed by economists are often more complex. For example, in college admissions markets, universities often offer admission to several study programs and multiple levels of financial aid. These more complex environments are studied in the matching-with-contracts literature (Hatfield and Milgrom, 2005). Much of this literature focuses on

[1]See Roth and Xing (1994) and Roth (2002).

[2]See Banerjee et al. (2013).

[3]A large portion of the literature on the theory of two-sided matching markets is motivated by the potential multiplicity of stable allocations. Examples include studies of the structure of the set of stable allocations (Knuth, 1976), of fair stable allocations (Teo and Sethuraman, 1998; Klaus and Klijn, 2006; Schwarz and Yenmez, 2011), of the extent to which it is possible to improve the allocation of under-demanded hospitals or to increase the number of assigned doctors (Roth, 1986), and of incentives (Roth, 1982; Sönmez, 1999; Ehlers and Massó, 2007).

[4]Unless otherwise specified, DA refers to the student-proposing version of DA (Gale and Shapley, 1962).

[5]Truthful reporting to the student-proposing DA mechanism is a weakly dominant strategy for students, and there is no stable matching mechanism that makes truthful reporting dominant for both sides of the market (Dubins and Freedman, 1981; Roth, 1982). Numerous studies have analyzed the optimal behavior of schools when the student-proposing DA mechanism is in place (examples include Sönmez, 1997; Roth and Rothblum, 1999; Ehlers, 2004; Konishi and Ünver, 2006; Coles, Gonczarowski and Shorrer, 2014). Azevedo and Budish (2012) show that in large markets it is safe to report one's true preferences to DA.

identifying conditions under which DA remains stable and strategy-proof for students. The motivating question of this paper is: *Do the findings on the size of the set of stable allocations and the good incentive properties persist in such environments?*

They do not. We study a natural extension of the Gale and Shapley (1962) matching market model (without contracts), which captures the structure of preferences observed in centralized college admissions markets.[6] We observe that under DA, financial aid decisions are based on merit (Theorem 1),[7] and thus ignore students' outside options. While several centralized college admissions clearinghouses use variants of DA and thus allocate financial aid based on merit, in all the instances we are aware of the mechanism was introduced prior to the introduction of multiple levels of aid, and we therefore do not know if the choice to allocate financial aid based on merit was intentional.

Based on this observation, we show that in large college admissions markets the expected fraction of students and colleges that have multiple stable allocations does not vanish as the size of the market grows large, and the same is true for the fraction of colleges that can successfully manipulate DA. Furthermore, the manipulation we identify takes a simple form that can be interpreted as colleges exercising their local market power over price-insensitive students by offering need-based – rather than merit-based – financial aid. Our proofs use a natural extension of the large (matching-without-contracts) market model of Kojima and Pathak (2009), but they have clear analogues in other models of large two-sided matching markets.

Empirically, we corroborate the predictions of the model using data from two centralized college admissions markets, where variants of DA are in use. We show, based on reported preferences, that in Hungary, thousands of students have multiple stable allocations. Moreover, we show that switching from the DA outcome to another stable allocation would increase the number of students accepted to colleges in this country by more than 3%, and that colleges could substantially improve the quality of their incoming cohorts by using different admissions criteria. Rural applicants and applicants from lower socioeconomic backgrounds would benefit disproportionately.

[6]The theory of matching with contracts has many other applications, such as the allocation of cadets to military branches (Sönmez, 2013; Sönmez and Switzer, 2013), school choice with dormitories or "special classes" (Wang and Zhou, 2018), and entry-level labor markets (Niederle, 2007; Dimakopoulos and Heller, 2015). While the focus of this paper is on college admissions markets, our results also apply to these other environments.

[7]We are grateful to Joel Sobel for suggesting this interpretation of our result.

Our findings suggest that answering "classic" questions in the theory of two-sided matching markets for the college admissions setting may be a fruitful direction. A natural question, for example, is: *How does one find the stable allocation that matches the most students?*

We begin with two examples illustrating the key ideas behind our main results. The first example shows that the presence of multiple contractual terms may leave room for bargaining between a college and a student, even when the outcome must be stable. In the example, stability is not compromised if a college refuses financial aid to a student whose choice of college is not sensitive to the availability of financial aid, and who thus has no attractive outside options.

The second example shows that when colleges face a budget constraint, the outcome of the bargaining with one student affects the identity and the quantity of the other students the college can recruit. In the example, by refusing financial aid to a price-insensitive students, the college relaxes its budget constraint, which allows it to recruit better, price-sensitive students. Our main theoretical result (Theorem 2) shows that there are many instances in which colleges can manipulate DA by reporting that price-insensitive students are ineligible for admission with financial aid, and shows that this manipulation results in a stable allocation that these colleges prefer. Intuitively, the reason is that under DA, financial aid decisions are based on merit, and thus ignore students' outside options. Readers who find this verbal description satisfactory may want to skip the examples and go directly to Section 1.2.

1.1 Examples

The following examples use the notation of the paper, but since the model has not yet been introduced, we provide a verbal description for each piece of notation.

Example 1. There are n colleges and m students. Colleges offer positions with or without financial aid (formally, $T = \{0, 1\}$). Each college has a capacity of one regardless of funding ($q_0 = q_1 = 1$), and it prefers all student and financial aid combinations to the outside option of staying unmatched. For all colleges, accepting a student, s, without financial aid is preferred to accepting the same student with financial aid. However, colleges care lexicographically more about the identity of the student than about funding (i.e., if accepting s without financial aid is preferred to accepting s' with financial aid, then accepting s with financial aid is also preferred to accepting s' with financial aid). Students prefer admission with financial aid, but they too care lexicographically more about the identity of the college than about funding. Students also prefer any allocation to staying unmatched.

Claim 1. *Under the above conditions, there are* $\min\{m, n\}$ *colleges and* $\min\{m, n\}$ *students with multiple stable allocations.*

Proof. Since colleges have strict preferences and unit demand, there exists a stable allocation and the same agents are matched in all stable allocations (Hatfield and Milgrom, 2005). Since all agents find any allocation acceptable, stability implies that there cannot be unmatched agents on both sides of the market. Hence, $\min\{m, n\}$ colleges and students have some stable allocation other than being unmatched. Given a stable allocation, changing the terms while maintaining the identities of the contracting parties preserves stability. $\square$

Note that even if colleges know nothing more than the above description about the preferences of students, they have a simple manipulation for the student-proposing version of DA: by declaring all funded contracts unacceptable and reporting their true preferences over unfunded contracts, colleges will be assigned the same students, but will not have to provide financial aid.[8] The driving force behind Example 1 is the near indifference of both sides of the market to funding, which implies that no agent has an attractive outside option. The requirement that all agents on the other side of the market be acceptable does not play an important role: in a model without this assumption, the only difference would be that $\min\{m, n\}$ is replaced with the cardinality of some stable allocation.

While the set of stable allocations in the market described in Example 1 is large in the sense that many agents (students and colleges) have multiple stable allocations, the proof relies on allocations in which the same agents contract with each other, and only the contractual terms differ. Given that we assumed that contractual terms are not particularly important for any of the agents (relative to the identity of their partner), the set of stable allocations may still be small in the sense that the same agents are matched

[8] This holds under the assumption that students will use their weakly dominant strategy of reporting their preferences truthfully.

in all stable allocations, and in the sense that agents do not have "strong" preferences between stable allocations.

In the following example with one college, students differ in the importance they attribute to financial aid, and the college can accept more students than it can fund. The example highlights several differences between our college admissions environment and the one studied by Gale and Shapley (1962), where colleges offer students only one package of contractual terms. Notably, in our environment there is no student-optimal stable allocation, and the number of students attending college differs between stable allocations.

Example 2. There is one college ($C = \{h\}$) with two seats, but only one scholarship available ($T = \{0, 1\}$, $q_h^0 = 2$, $q_h^1 = 1$), and two students, $S = \{r, p\}$. It may help to think of r (she) as a "rich" applicant and p (he) as a "poor" applicant. In subsequent examples h will be the "special" college whose perspective we take, and other colleges will be denoted by c.

The rich applicant, r, also happens to be a better "fit" with college h ($r \gg_h p$). The college prefers to accept the best "fit" students, and to fill its capacity. The college's preferences over acceptable allocations are summarized as follows:

$$\{(r,h,0),(p,h,0)\} \succ_h \{(r,h,1),(p,h,0)\} \succ_h \{(r,h,0),(p,h,1)\} \succ_h$$

$$\succ_h \{(r,h,0)\} \succ_h \{(r,h,1)\} \succ_h \{(p,h,0)\} \succ_h \{(p,h,1)\} \succ_h \emptyset,$$

where 1 (0) indicates admission with (without) financial aid. Applicant r's preferences are $(r,h,1) \succ_r (r,h,0) \succ_r \emptyset$. That is, she prefers to receive financial aid, but is willing to attend h even if she does not get it. The poor applicant, p, is only interested in admission with financial aid. Thus, his preferences are summarized by $(p,h,1) \succ_p \emptyset$. Under these preferences, there are two stable allocations, $\{(r,h,1)\}$, which is the result of the student-proposing DA algorithm, and $\{(r,h,0),(p,h,1)\}$.

Notably, the two allocations have different numbers of assigned students. Moreover, the outcome of DA is not the stable allocation most preferred by both students. In fact, a student-optimal stable allocation does not exist.[9]

[9] To see that there is no stable allocation that is most preferred by both students, note that the only allocation that both students weakly prefer to the above-mentioned stable allocations has both of them receiving financial aid, but this allocation is not acceptable to the college.

1.2 Overview of the results

Our model of college admissions markets is a natural extension of the Kojima and Pathak (2009) model of large two-sided matching markets (without contracts). In their model, while the number of schools is large, each student finds a small number of them acceptable. We augment their model by introducing different levels of financial aid from the same college. We assume that colleges face a constraint on the number of financial aid packages at each level, but otherwise have no strong preferences over the amount of financial aid they provide and over the identities of the funded students in a given cohort. We also assume that whenever an applicant finds a certain college acceptable under some terms, she prefers to attend that college under a more generous financial aid package.

The assumption on colleges' preferences (which we relax substantially in Appendix D) reflects the reports of departments participating in the Israeli Psychology Master's Match (IPMM; Hassidim, Romm and Shorrer, 2017), and is consistent with the choice functions used by Hungarian colleges (Biró, 2012). The assumptions on students' preferences reflect many features of the preferences reported to two centralized college-admissions matching-with-contracts markets: the IPMM and Hungarian college admissions (Biró, 2012).[10] We show in Appendix C that the assumption that students always prefer more generous financial aid could be substantially relaxed as well, but we make it to ensure that our results do not rely on configurations of preferences that we find unreasonable in many applications. Additionally, our results have clear analogues in other models of large matching markets (without contracts) where applicants are not assumed to be interested only in a limited number of schools (e.g., Ashlagi, Kanoria and Leshno, 2017; Azevedo and Leshno, 2016).

We prove that the expected fractions of applicants and of colleges whose assignment is different across stable allocations are large (non-vanishing in large markets), and that the same holds for the fraction of colleges that can successfully manipulate DA when all other agents are truthful. Furthermore, different stable allocations may result in substantially different numbers of

[10]The IPMM is strategy-proof for applicants. Like most real-life implementations of DA, the Hungarian college admissions mechanism is not, strictly speaking, strategy-proof (Pathak and Sönmez, 2013; Shorrer and Sóvágó, 2017). Outside of the matching literature, Avery and Hoxby (2004) make similar assumptions on students' preferences. They too find empirically that students apply to a limited number of colleges.

students admitted to college.

We corroborate our theoretical predictions using administrative data from both the IPMM and the Hungarian market. We find that in Hungary, choosing a different stable allocation would increase the number of students admitted to college under the current student-proposing algorithm (about 60,000 a year) by more than 3%. Applicants from a lower socioeconomic background would benefit disproportionately,[11] as would female and rural applicants. Moreover, colleges could successfully manipulate DA and "poach" talent from their competitors. Our findings stand in sharp contrast to those of Roth and Peranson (1999), who find that only about 0.1% of approximately 20,000 applicants to the National Residency Match Program (NRMP) in the early 1990s would have received a different assignment had the algorithm been changed from hospital-proposing to applicant-proposing.

To provide intuition, we highlight the main differences between our model and that of Kojima and Pathak (2009), who find that under DA, when all agents are truthful, schools' incentives to misrepresent their preferences are minimal, and the set of stable allocations is small. Kojima and Pathak attribute their results to the *"vanishing market power"* of schools when the student-proposing version of DA is used. Namely, even knowing others' actions, a school is highly unlikely to be able to strategically reject a student and as a result receive a proposal from another, preferred student, a necessary condition for the existence of a profitable manipulation in their setting.

By contrast, the driving force behind our results is the presence of *market power*. Given a stable allocation, colleges have *local* market power over students who receive financial aid, but who have no outside option that they prefer to a contract with the same college at a lower level of financial aid. An extreme case is when students rank all contracts with the same college consecutively. In this case, the college can "price discriminate" by offering the lowest acceptable level of financial aid to such students (as illustrated in Example 1), and the freed-up funds can then be used to recruit price-sensitive students (as illustrated in Example 2). Since students' outside options depend on their preferences and on the behavior of other agents (students and colleges), information on others is crucial for colleges in order to successfully implement this strategy.

The presence of market power in our large-market model is the result of

[11] Intuitively, the reason is that under DA financial aid is allocated based on merit (as illustrated by Example 2).

colleges offering several contracts that the same students tend to be interested in. In contrast to the environment without contracts studied in Kojima and Pathak (2009), colleges can successfully manipulate DA without generating offers from additional students. It is sufficient that a strategically rejected student apply for admission with a lower level of financial aid, as this would relax the college's budget constraint, and allow it to accept a price-sensitive student it would have otherwise had to reject.

Our results shed light on the policy debate around market power in higher education (e.g., Hoxby, 2000). This literature gained traction after the Department of Justice brought an antitrust case against a group of elite colleges for sharing prospective students' financial information and coordinating their financial aid policy. MIT contested the charges, claiming that this practice prevents bidding wars over the best students and thus frees up funds to support needy students, and that MIT does not profit financially from this practice (DePalma, 1992). In 1994, Congress passed the Improving America's Schools Act, whose section 568 permits some coordination and the sharing of information between institutions with a need-blind admissions policy. Our model provides theoretical support to MIT's arguments. We show that even in the absence of a motive to increase profit, colleges have an incentive to apply market power in order to improve the quality of their incoming cohorts, and that the consequences for students are heterogeneous. In particular, the direct effect is that some (needy) students gain and other (wealthy) students lose. Furthermore, the model illustrates how information about students' alternatives can facilitate such behavior.[12]

Finally, we make two smaller contributions. First, we show that, in contrast to the environment studied by Gale and Shapley (1962), in our environment a college may have multiple stable allocations but not be able to manipulate DA. Second, we show that while DA always terminates in a stable allocation in college admissions markets, the class of college admissions markets cannot be embedded (in the sense of Echenique, 2012) in Kelso and

[12] More recently, in 2013, a session in the annual winter meeting of the Council of Independent Colleges on how to end bidding wars for students and curtail the use of generous merit-aid packages led to a new DOJ investigation (which was subsequently closed). The National Association of Independent Colleges and Universities has recommended to Congress to consider providing a temporary antitrust exemption for private, nonprofit colleges, allowing them to share information and coordinate their financial aid policies. For more details see: `https://www.naicu.edu/policy-advocacy/student-aid/anti-trust`.

Crawford 1982 matching-with-salaries markets.

1.3 Related literature

Our study is most closely related to papers studying the size of the set of stable allocations in two-sided matching markets. There are many ways to think about the size of this set, and thus multiple notions of "smallness." One such notion is where the same agents are matched in all stable allocations. This result is part of the rural hospital theorem proved by Roth (1984a, 1986) for the case of many-to-one markets with responsive preferences, and extended by Hatfield and Milgrom (2005) to environments with contracts where programs' preferences meet certain conditions.

An alternative notion of smallness has to do with the number of agents who receive a different assignment in different stable allocations. Early studies focus on marriage markets with the same number of men and women, where all members of the opposite sex are acceptable. In a random instance of such a market where strict preferences are drawn independently and uniformly at random, the set of stable allocations is typically large in this sense (Pittel, 1989, 1992).

Roth and Peranson (1999) show that the set of stable allocations in the NRMP is small in this sense.[13] They attribute their finding to the market being large and students ranking only a small number of residency programs, and provide simulation evidence in support of their theory. This finding, which Roth and Peranson (1999) refer to as "core convergence," is later proved theoretically by Immorlica and Mahdian (2015), Kojima and Pathak (2009), and Storms (2013), in increasingly general environments. Under an additional regularity condition, Kojima and Pathak (2009) prove that truthful reporting to DA is an approximate Bayesian Nash equilibrium. Ashlagi, Kanoria and Leshno (2017) show that the set of stable allocations is typically small even when all members of the opposite sex are acceptable, as long as the number of men and women is not exactly equal. Azevedo and Leshno (2016) study a model with a finite number of schools and a continuum of students, and find that generically there is a unique stable allocation.

[13] Similar results are found by Banerjee et al. (2013) in Indian marriage markets, and by Hitsch, Hortaçsu and Ariely (2010) in an online dating market. Menzel (2015) shows that in large markets the set of stable allocations converges in the sense that the probability of a man of a certain type being matched with a woman of a certain type converges to a unique limit.

Another notion of smallness is that the difference in utility between different stable allocations is small for all (or most) agents. Holzman and Samet (2014) find that the set of stable allocations in marriage markets is small in this sense when preferences are correlated. Lee (2016) allows for correlation in preferences through a common-value component and finds that under certain conditions on preferences the set of stable allocations is small, and so are incentives to misreport under DA in marriage markets.[14]

In Section 3 we show that the set of stable allocations of large college admissions markets is large relative to all three of the above-mentioned senses of smallness. In Appendix A, we show that the set of stable allocations in college admissions markets coincides with the weak-domination core, which is contained in the (strict-domination) core. Thus, we show that the core in college admissions markets does not "converge."

Our findings complement those of Azevedo (2014), who studies competition in quantities (i.e., capacity) in the Azevedo and Leshno (2016) environment. Azevedo (2014) finds that when schools are small compared to the rest of the market, they have no incentive to reduce their capacities. When schools are large, they have an incentive to exercise their market power by reducing their capacities. In the environment that we study, colleges' preferences and strategy spaces are more complex, and colleges are able to exercise *local* market power over interested students, even when their size is negligible relative to the rest of the market. Furthermore, the manipulations we study may *increase* the number of students assigned to the manipulating college.

Our paper is related to the growing literature on matching with contracts (e.g., Kelso and Crawford, 1982; Roth, 1984b; Fleiner, 2003; Hatfield and Milgrom, 2005; Hatfield and Kojima, 2010; Hatfield and Kominers, 2015; Hatfield, Kominers and Westkamp, 2015), which has been applied to study other questions related to college admissions (Abizada, 2016; Afacan, 2017; Aygün and Bó, 2016; Pakzad-Hurson, 2014; Westkamp, 2013; Yenmez, 2018). Wang and Zhou (2018) conduct an empirical study of high-school choice in China, where the (manipulable) mechanism in place offers students the ability to pay higher tuition in exchange for admission priority to a fraction of the seats that each school offers.

Closely related to our paper are studies of reserve design in the context of school choice (Dur et al., 2013; Dur, Pathak and Sönmez, 2016). A key

[14] Coles and Shorrer (2014) show that even under incomplete information the exact best response of schools under DA can be substantially different from truthful reporting.

observation in this literature is that applicants are indifferent between different seats in the same school, which implies, using our terminology, that schools have market power over all assigned students. The reserve-design literature focuses on the effect of different ways the mechanism can break these preference ties to form strict student rankings of contracts, while keeping the priorities at each seat fixed. By contrast, we study an environment where students have strict preferences over all contracts, and we concentrate on changes to colleges' preferences.

In the simple one-to-one setting, a school has an incentive to misreport its preferences to the student-proposing DA mechanism if and only if the school has multiple stable allocations (Demange, Gale and Sotomayor, 1986). In many-to-one markets, only one implication is correct (e.g., Kojima and Pathak, 2009). Namely, given a profile of preferences, a school may have a unique stable allocation and still have an incentive to misrepresent its preferences to a DA mechanism. We show that in college admissions markets neither statement implies the other. Namely, unlike a school, a college may also have multiple stable allocations but no incentive to misrepresent its preferences to a DA mechanism.

Echenique (2012) shows that when colleges' preferences satisfy the substitutes condition of Hatfield and Milgrom (2005), there exists an embedding that injectively maps markets with contracts to Kelso and Crawford (1982) markets with salaries and gross substitutes demands, such that the set of stable allocations is preserved.[15] The construction does not apply to environments like the one studied by Sönmez and Switzer (2013), where only the weaker condition of unilateral substitutability (Hatfield and Kojima, 2010) is satisfied. Schlegel (2015) and Jagadeesan (2016) construct different embeddings that extend to this broader class of markets. We contribute to this literature by identifying a real-life environment where DA is stable and strategy-proof for students, but cannot be embedded in a Kelso and Crawford environment. To see this, note that the college admissions market in Example 2 does not have a student-optimal stable allocation, a necessary condition for the existence of an embedding.

The paper is organized as follows. Section 2 presents our model and discusses the differences between our college admissions environment and the environment studied by Gale and Shapley (1962). Section 3 presents the main theoretical result, and a proof of a special case that illustrates key

[15]Kominers (2012) extends this result to the many-to-many setting.

ideas in the complete proof (which is in Appendix B). Section 4 presents the empirical evidence. Section 5 discusses the implications of our findings and proposes directions for future research.

2 Model

We use the many-to-one matching-with-contracts model of Hatfield and Milgrom to describe college admissions environments. There is a finite set of *colleges*, C, a finite set of *students*, S, and a finite set of *contractual terms*, T. A *contract* is a tuple $(s, c, t) \in S \times C \times T$ that specifies a student, a college, and the contractual terms that govern their relationship. In this paper, $t \in T$ will typically describe the level of financial aid. The set of all possible contracts, X, is a subset of $S \times C \times T$.

We denote by X_i the set of all possible contracts that involve agent $i \in S \cup C$. Each agent, i, has strict preferences over subsets of X_i, which we denote by $\succ_i$. We often follow the convention of omitting sets that are ranked lower than the empty set from the description of $\succ_i$.

An *allocation* is a subset $Y \subseteq X$. Given an allocation Y, we sometimes refer to $Y_i := Y \cap X_i$ as agent i's allocation. An allocation Y is *individually rational* if for any agent i the entire set Y_i is the most-preferred subset of Y_i.

Finally, we assume that all students prefer the empty set to any subset with cardinality strictly greater than 1. Given that our interest is in individually rational allocations only, this encodes our assumption that the market is a many-to-one matching market. An allocation Y is *feasible* if $|Y_s| \leq 1$ for each student s.

Financial aid

We assume that T is a finite subset of $\mathbb{N}$. Unless otherwise specified, we think of $t \in T$ as a funding level, and assume that for each student, s, and college, c, $(s, c, t) \succ_s (s, c, t')$ if and only if $t > t'$. As we discuss later, this modeling choice is not crucial for any of our results, but is made in order to capture key aspects of our datasets.

We make several assumptions on colleges' preferences. First, each college, c, is associated with a sequence of numbers, $\{q_t^c\}_{t \in T}$, such that if $t < t'$ then $q_t^c \geq q_{t'}^c$. The number q_t^c represents a constraint on the number of students who can be accepted with funding level t or higher. Each college, c, prefers the empty allocation to all allocations that *violate c's quotas*, that is, that assign to c more than q_t^c students with a funding level t or higher.[16]

Second, each college, c, has a *master list*, a complete order over $S \cup \{\emptyset\}$, denoted by $\gg_c$. Given an allocation Y, a contract $(s, c, t) \in Y_c$, and a contract $(s', c, t) \notin Y_c$, $Y_c \succ_c Y_c \setminus (s, c, t) \cup (s', c, t)$ if and only if $s \gg_c s'$. Moreover, if Y_c does not violate c's quotas, $Y_c \succ_c Y_c \setminus (s, c, t)$ if and only if $s \gg_c \emptyset$. In words, the college will accept an additional contract as long as the student is ranked higher than the empty set on the college's master list, and the contractual terms will not cause a quota violation.

Finally, we assume that, as long as quotas are not exceeded, the allocation of funding is less important than the identities of incoming students. Formally, if $Y_c \succ_c \emptyset$ and $Y'_c \succ_c \emptyset$, and there exists a bijection $\phi : Y_c \to Y'_c$ such that $x \in X_s \iff \phi(x) \in X_s$ for all $x \in Y_c$ and all $s \in S$ (i.e., Y_c and Y'_c differ only in contractual terms), but no such bijection exists between Y_c and Y''_c (roughly, Y_c and Y''_c differ in contracting parties), then $Y''_c \succ_c Y_c \iff Y''_c \succ_c Y'_c$. This assumption, which we relax substantially in Appendix D, is realistic insofar as funding terms and quotas are often exogenously given.[17]

The special case of our model where $|T| = 1$ corresponds to two-sided many-to-one matching-without-contracts markets with responsive preferences, the environment studied by Kojima and Pathak (2009).

Choice functions and stability

Agent i's preferences induce a *choice function*, $\text{Ch}_i : 2^X \to 2^{X_i}$, that identifies the subset of Y_i most preferred by i for any subset Y of X. Formally, $\text{Ch}_i(Y) := \max_{\succ_i} Z \subset Y_i$.[18] An allocation Y is *unblocked* if there does not exist a college, c, and a non-empty $Z \subset X_c \setminus Y$ such that $Z_i \subset \text{Ch}_i(Z_i \cup Y_i)$ for all $i \in S \cup C$. An allocation Y is *stable* if it is individually rational and

[16] In the language of NRMP, we allow for "reversions," i.e., donations of unfilled positions from one program to another (Roth and Peranson, 1999; Niederle, 2007). All of our results hold in an alternative model in which each college has a constant capacity for each level of funding, but seats at one level cannot be converted to seats at other level.

[17] For example, funding may come from the government in the form of a tuition waiver (see Artemov, Che and He, 2017). Alternatively, the "college" in our model may represent a department that is free to make admissions decisions, but whose funding policy is set by the institution and whose budget is earmarked (see Hassidim, Romm and Shorrer, 2017).

[18] The fact that choice functions are derived from strict preferences implies that they satisfy the irrelevance-of-rejected-contracts condition (Aygün and Sönmez, 2013).

unblocked. In Appendix A we show that an allocation is stable if and only if it belongs to the weak-domination core, which is a subset of the (strict-domination) core (Roth and Sotomayor, 1990).

A remark on our choice to make assumptions directly on preferences, rather than on choice functions, is in order. It is well known that, in general, assumptions on choice functions can be less restrictive than assumptions on preferences. We choose to make assumptions directly on preferences since we think that this makes them more transparent. These assumptions also reflect our understanding of colleges' preferences, based on our practical experience (e.g., Hassidim, Romm and Shorrer, 2017).

2.1 Properties of college admissions environments

We begin this section by noting that we have not restricted colleges' choice functions to feasible allocations (i.e., at most one contract per student). With this observation, it is easy to verify that the choice functions satisfy the hidden-substitutes condition of Hatfield and Kominers (2015), and that they meet the other conditions of their Theorems 1–3 that assure that DA yields a stable allocation and is strategy-proof for students.

Observation 1. *The set of stable allocations is non-empty in college admissions environments. Furthermore, DA terminates in a stable allocation, and the mechanism it induces is strategy-proof for students.*

Observation 1 lists some similarities between our model and the matching-without-contracts environment studied by Gale and Shapley. Critically for our main result, it guarantees the existence of a stable allocation and assures that DA terminates in one such allocation.

The following theorem formalizes our claim that under DA financial aid is distributed based on merit.

Theorem 1. *Let* $\left\langle S, C, T, \{\succ_c, \gg_c, q_t^c\}_{t\in T}^{c\in C}, \{\succ_s\}_{s\in S}\right\rangle$ *be a college admissions market, and let* Y *be the stable allocation that corresponds to the outcome of DA. If two students are assigned to the same college under* Y*, then the one the college ranks higher receives (weakly) more financial aid. Formally, for all* s *and* s' *in* S*, for all* c *in* C*, and for all* t *and* t' *in* T*, if* (s, c, t) *and* (s', c, t') *belong to* Y*, then* $s \gg_c s'$ *implies* $t \geq t'$*.*

Proof. Assume to the contrary that $s \gg_c s'$ but $t < t'$. Since students prefer higher levels of financial aid, it must be the case that s requested admission to

c under t', and his offer was rejected before the algorithm terminated. At the point where this rejection occurred, c tentatively held contracts with financial aid levels t' and higher from $q^c_{t'}$ students who were all ranked higher than s. But since $s \gg_c s'$, this implies that a request of s' for admission under t' must be rejected before the termination of the algorithm, a contradiction. □

We next highlight key differences between our model and the matching-without-contracts environment studied by Gale and Shapley (1962).

Proposition 1. *A college admissions market may have no stable allocation that is most preferred by all students. Furthermore, different stable allocations may have different numbers of assigned students.*

Proof. Follows from Example 2 in Section 1.1. □

Corollary 1. The class of college admissions markets cannot be embedded (in the sense of Echenique, 2012) in Kelso–Crawford markets with salaries and gross substitutes demands.

The corollary follows by the fact that the existence of a student optimal stable allocation is a necessary condition for an embedding to exist.

Proposition 2. *i) In a college admissions market, a college may have multiple stable allocations and yet not be able to manipulate DA when all other agents are truthful. ii) Furthermore, it may have a unique stable allocation and yet be able to manipulate DA when all other agents are truthful.*

Proof. The second part holds even in the absence of contracts (Kojima and Pathak, 2009). The first part follows from Example 3. □

Example 3. We add to the (special) college h from Example 2 another (community) college, c, with one seat and no scholarships available ($C = \{h, c\}$, $q^c_0 = 1$, $q^c_1 = 0$). The poor student's first choice is the funded seat at h, and although he now prefers the unfunded seat at h to staying unmatched, he finds the (cheaper) community college more attractive. The rich student prefers college h to the community college under any funding terms.

Formally, the students' preferences are now

$$(r, h, 1) \succ_r (r, h, 0) \succ_r (r, c, 1) \succ_r (r, c, 0) \succ_r \emptyset, \text{ and}$$

$$(p, h, 1) \succ_p (p, c, 1) \succ_p (p, c, 0) \succ_p (p, h, 0) \succ_p \emptyset,$$

c's preferences are

$$(r, c, 0) \succ_c (p, c, 0) \succ_c \emptyset,$$

and h's preferences remain

$$\{(r, h, 0), (p, h, 0)\} \succ_h \{(r, h, 1), (p, h, 0)\} \succ_h \{(r, h, 0), (p, h, 1)\} \succ_h$$

$$\succ_h \{(r, h, 0)\} \succ_h \{(r, h, 1)\} \succ_h \{(p, h, 0)\} \succ_h \{(p, h, 1)\} \succ_h \emptyset.$$

There are two stable allocations: $\{(r, h, 1), (p, c, 0)\}$, which is the result of the student-proposing DA, and $\{(r, h, 0), (p, h, 1)\}$. The number of students attending each college is different between the two stable allocations. Again, the outcome of DA is not the stable allocation most preferred by all students. And the only allocation that both students weakly prefer to both the above allocations requires both students to be funded by the special college, which violates the college's individual rationality constraint. It is easy to verify that, when all other agents are truthful, c cannot do better than the outcome of DA, as r must be assigned to h regardless of c's strategy.

The fact that c cannot manipulate DA even though it has multiple stable allocations is not due to one of them being the empty allocation. We could have augmented the example by adding a third student, d, that is only interested in the community college, and whom all colleges least prefer. The stable outcomes would be $\{(r, h, 1), (p, c, 0)\}$, which is the result of the student-proposing DA, and $\{(r, h, 0), (p, h, 1), (d, c, 0)\}$, but c would still have no incentive to misrepresent its preferences to DA when others are truthful.

The example also illustrates the role of outside options. The special college, h, has market power over the rich student in the allocation that results from DA because she has no outside option that she prefers to the unfunded contract with h. The special college does not have market power over the poor student in the second stable allocation since, if he is not offered funding, he prefers to attend another college that will accept him. Example 4 in Appendix E shows that given a stable allocation, a college may have market power over an assigned student who ranks other contracts between that student's allocation and another contract with the college, as long as the colleges who are parties to these contracts are not interested (and thus the student cannot form a blocking coalition with them). This observation proves useful for our empirical analysis.

It is worth pointing out that in the previous example, if the student-proposing version of DA is used, the private college has a simple profitable manipulation of declaring that the rich student is not eligible for financial aid. This manipulation is also feasible in a large market where the wealth level of applicants is known, and nothing else is known about students' preferences other than that financial aid does not alter rich applicants' preferences between colleges.

3 Theoretical Evidence

By now it is clear that college admissions environments have some properties that distinguish them from markets without contracts. We now turn to address our main questions: *How likely can a college manipulate the student-proposing DA mechanism*, and *how likely does a college (student) have multiple stable allocations?* To this end, we introduce a random environment that generalizes the Kojima and Pathak (2009) model of large matching markets to college admissions environments.

3.1 Regular sequences of college admissions markets

Let a *uniform random market* be a tuple $\tilde{\Gamma} = \left\langle S, C, T, \{\succ_c, \gg_c, q_t^c\}_{t \in T}^{c \in C}, k \right\rangle$, where k is an integer greater than one, and $\succ_c$ represents college c's strict preferences (which must be consistent with its master list, $\gg_c$, and list of quotas, $\{q_t^c\}_{t \in T}$). A uniform random market induces a college admissions market by drawing students' preferences randomly in the following way:

- Step 1: for each student independently, draw k different colleges from C uniformly at random.
- Step 2: for each student independently, draw uniformly at random an acceptable permutation over the $k \times |T|$ possible contracts with the colleges that were drawn in Step 1, where an acceptable permutation satisfies our assumption that students prefer higher levels of funding. Set the realized permutation as the student's preferences. Other contracts are not acceptable to the student.

For each realization of students' preferences, a (non-random) college admissions market is obtained. Setting $|T| = 1$ yields the distribution of preferences in uniform Kojima–Pathak markets. For notational convenience, we maintain the assumption of *uniform* random markets until the end of this section. In Appendix C we show that our results continue to hold in a much broader class of preference distributions (Proposition 4). The natural generalization of the preference structure studied in Kojima and Pathak (2009), which allows for correlation between students' preferences, falls within this class, as do cases where contracts are not naturally ranked (e.g., multiple study tracks),[19] or where some fraction of the population considers certain levels of financial aid prohibitively low. In Appendix D we show that our results continue to hold when colleges care about the identity of the recipients of financial aid, as long as this consideration does not always supersede the quality of these recipients.

A *sequence of uniform random markets*, denoted by $\{\tilde{\Gamma}^n\}_{n=1}^{\infty}$ where $\tilde{\Gamma}^n := \left\langle S^n, C^n, T^n, \{\succ_c, \gg_c, q_t^c\}_{t\in T^n}^{c\in C^n}, k^n \right\rangle$, is *regular* if there exist integers k,l,$\bar{q}$, and λ, all greater than one, such that:

1. $|C^n| = n$ for all n,
2. $k^n = k$ and $T^n = \{0, 1, ..., l-1\}$ for all n,
3. $q_0^c \leq \bar{q}$ for all $c \in C^n$ and all n,
4. for all n, $c \in C^n$, and $s \in S^n$, $s \gg_c \emptyset$,
5. for all n and $c \in C^n$, there exist $t, t' \in T^n$ such that $q_t^c > q_{t'}^c > 0$, and
6. $\frac{1}{\lambda} n \leq |S^n| \leq \lambda n$, for all n.

Condition 1 assures that the number of colleges grows as the sequence progresses. Condition 2 assures that the number of contracts that students consider acceptable is uniformly bounded on the sequence. Condition 3 assures that the number of positions in each college is uniformly bounded across colleges and markets. Condition 4 assures that colleges find any student acceptable. These conditions are identical to those of Kojima and Pathak (2009).

Condition 5 is the key addition we make to their model. This condition assures, roughly, that each college faces a financial aid constraint on top of the capacity constraint in Condition 3. Finally, Condition 6 assures that

[19] This generalization also captures centralized markets where contracts are naturally ranked but some applicants make mistakes (Hassidim, Romm and Shorrer, 2016; Artemov, Che and He, 2017; Shorrer and Sóvágó, 2017).

the number of students does not grow much faster or much slower than the number of colleges. Kojima and Pathak (2009) require only the first half of this condition. We require the second half as well since we are interested in instances where a substantial fraction of colleges have multiple stable allocations, but in markets with a small number of students (who each find contracts with at most k colleges acceptable) most colleges will not even be a party to any individually rational allocation. Omitting both the lower bound on $|S^n|$ and Condition 5 and requiring $|T| = 1$ yields the definition of a sequence of uniform Kojima–Pathak markets.

3.2 Main theoretical results

Consider a regular sequence of uniform random markets, $\{\tilde{\Gamma}^n\}_{n=1}^{\infty}$. For the n-th market, let $\alpha(n)$ denote the expected number of students with multiple stable allocations, let $\beta(n)$ denote the expected number of colleges with multiple stable allocations that can successfully manipulate DA when all other agents are truthful, let $\gamma(n)$ denote the expected number of colleges with multiple stable allocations that cannot successfully manipulate DA when all other agents are truthful, let $\delta(n)$ denote the expected number of colleges with different numbers of assigned students in different stable allocations, and let $\eta(n)$ denote the expected number of students who are matched in some stable allocation, but are unmatched in another; for simplicity, our notation suppresses the dependence on the sequence.

Theorem 2. *Given a regular sequence of uniform random markets, there exists $\Delta > 0$ such that:*

1. $\liminf_{n\to\infty} \alpha(n)/n > \Delta$,
2. $\liminf_{n\to\infty} \beta(n)/n > \Delta$,
3. $\liminf_{n\to\infty} \gamma(n)/n > \Delta$,
4. $\liminf_{n\to\infty} \delta(n)/n > \Delta$, *and*
5. $\liminf_{n\to\infty} \eta(n)/n > \Delta$.

To keep the focus on the crux of the argument we defer the detailed proof to Appendix B and in what follows we analyze the special case of $T^n \equiv \{0, 1\}$

(e.g., students are either fully funded or not funded), $q_0^c = 2$ and $q_1^c = 1$ for all $c \in C^n$ for all n (i.e., each college has two seats and can offer one scholarship), and $|S^n| = 2|C^n|$ (i.e., there are as many students as there are college seats). We also defer the treatment of $\eta(n)$, which requires a more subtle argument.

Consider two students, $r, p \in S^n$, a college $h \in C^n$, and some other college $c \in C^n$. Let the event $E^n(r, p, h, c)$ denote the case where:

1. College h ranks r higher than p on its master list. Formally, $r \gg_h p$.

2. The only students who find contracts with h acceptable are r and p. Formally, for all $s \in S^n \setminus \{r, p\}$ and all $t \in T^n$, $\emptyset \succ_s (s, h, t)$.

3. The only student who finds contracts with c acceptable is p. Formally, for all $s \in S^n \setminus \{p\}$ and all $t \in T^n$, $\emptyset \succ_s (s, c, t)$.

4. The two contracts r finds most desirable are $(r, h, 1)$ and $(r, h, 0)$, and r prefers the first to the second. Formally, for all $z \in X_r \setminus X_h$, $(r, h, 1) \succ_r (r, h, 0) \succ_r z$.

5. The two contracts p finds most desirable are $(p, h, 1)$ and $(p, c, 1)$, and p prefers the first to the second. Formally, for all $z \in X_p \setminus \{(p, h, 1), (p, c, 1)\}$, $(p, h, 1) \succ_p (p, c, 1) \succ_p z$.

Note that in the event $E^n(r, p, h, c)$, the students r and p and the colleges h and c have two stable allocations: $\{(r, h, 1), (p, c, 1)\}$ (which is their allocation in the outcome of DA with respect to the true preference profile), and $\{(r, h, 0), (p, h, 1)\}$. Also, note that college h can successfully manipulate DA by declaring that allocations under which r receives financial aid are not acceptable, but clearly college c cannot do likewise. Furthermore, the two colleges have different numbers of students assigned to them in different stable allocations.

Lemma 1. *For any college $h \in C^n$, let $E_h^n = \bigcup_{(r,p,c) \in S^n \times S^n \times C^n} E^n(r, p, h, c)$. Then*

$$\liminf_{n \to \infty} \Pr\left[E_h^n\right] \geq \frac{2(k-1)}{k^2} \cdot e^{-4k}.$$

Proof. Given h, for different selections of (r, p, c), the events $E^n(r, p, h, c)$ are disjoint. There are $2n \cdot (2n-1) \cdot (n-1)$ possible selections such that $r \neq p$

and $c \neq h$. Half of these events have zero probability (when $p \gg_h r$). The probability of each of the other events is greater than

$$\underbrace{\left(1-\frac{k}{n-1}\right)^{4n}} \times \underbrace{\frac{1}{kn}} \times \underbrace{\frac{1}{n}\cdot\left(1-\frac{1}{k}\right)\cdot\frac{1}{n}} = \frac{k-1}{k^2n^3}\cdot\left(1-\frac{k}{n-1}\right)^{4n},$$

where each term in the leftmost expression corresponds to an (independent)[20] requirement from the definition of $E^n(r,p,h,c)$. Thus, the probability of the event E_h^n is greater than

$$n\cdot(2n-1)\cdot(n-1)\frac{k-1}{k^2n^3}\cdot\left(1-\frac{k}{n-1}\right)^{4n} \xrightarrow[n\to\infty]{} \frac{2(k-1)}{k^2}\cdot e^{-4k}.$$

□

The results on $\beta(n)$ and $\delta(n)$ follow immediately. The result on $\gamma(n)$ follows from a similar argument, where c is held fixed and the union is taken over selections of r, p, and h. The result on $\alpha(n)$ follows from the result on $\delta(n)$ and from the fact that each student finds contracts with at most k colleges acceptable.

4 Empirical Evidence

In this section we study the Hungarian college admissions market. We provide evidence in support of our assumptions on the demand structure (i.e., students' rank-order lists), corroborate the predictions of our model, and assess the potential welfare implications. Appendix F contains additional empirical evidence from the Israeli Psychology Master's Match.

4.1 Background

College admissions in Hungary have been controlled centrally and organized through a centralized clearinghouse since 1985.[21] Each year, about 100,000 students apply to bachelor's programs and approximately 60,000 are

[20]To be precise, the first term refers to both the second and third requirements, as they are not independent.

[21]For more details see Biró (2012) and references therein.

assigned. As is standard in Europe, prospective students must choose in advance a particular study program: a specific major at a specific institution (e.g., B.A. in applied economics at Corvinus University).

Citizens of Hungary and of other member states of the European Economic Area are eligible to receive up to six years (12 semesters) of state-funded education. However, the government limits the number of state-funded seats in each field of study. While only eligible applicants may apply for admission with state funding, unfunded positions are also available and are open to all.[22]

Over the years, the mechanism used by the clearinghouse has changed several times. Since 2008, a variant of student-proposing DA has been in use (Biró, 2012). Prior to that, a variant of the college-proposing algorithm was in place.[23] Both mechanisms endow applicants with field-specific priority scores based on a weighted average of several variables (mainly matriculation exam scores and GPA in the 11th and 12th grades, but also some credit for disabled, disadvantaged, or gifted applicants). The weights in the formula differ for different fields of study.

Since decisions about financial aid and admission are made simultaneously, and as the availability of funding may play a critical role in applicants' decisions between programs, applicants are allowed to submit rank-order lists (ROLs) ranking any number of contracts (program and funding-level combinations). For example, an applicant may submit an ROL that ranks three contracts with two programs: 1) a funded B.A. in applied economics at Corvinus University, 2) a funded B.Sc. in agricultural engineering at the University of Debrecen, and 3) an unfunded B.A. in applied economics at Corvinus University. Applicants who wish to submit an ROL that ranks more than three programs (corresponding to up to six contracts) are required to pay a fee (about $7 per additional program on the ROL). This feature implies that truthful reporting is not a dominant strategy in the Hungarian mechanism (Haeringer and Klijn, 2009). But given a set of programs on a list, it is dominated not to rank *all* acceptable contracts with these programs *truthfully*.

[22] In 2013, tuition ranged from $2,000 to $23,000 for three years, with a mean (over programs) of $4,500 and a median of $3,800. Many institutions grant funded students priority in access to subsidized housing and other amenities. The per capita GDP of Hungary in 2013 was $10,300.

[23] Under the college-proposing version of DA, each slot is regarded as a separate program and the program-proposing variant of DA is used with different slots in the same college using the same ranking over students.

To be clear, programs do not give precedence to applicants who apply for particular funding terms; they choose the highest-priority students available to them subject to the capacity and funding constraints.

After the match results are realized, applicants are informed of their placement, and the *priority-score cutoff* for each contract is made public. The priority-score cutoff for a contract is equal to the score of the lowest-scoring student was assigned to the particular contract.[24] These statistics receive extensive media coverage in the days after the match results are published.

4.2 Data

Our data on the Hungarian college admissions process between 2009 and 2011 is based on the dataset compiled by Shorrer and Sóvágó (2017), which merges data from four different sources. The main source of data is an administrative dataset containing much of the information available to the clearinghouse.[25] This dataset includes each applicant's ROL,[26] the priority score in each relevant contract, as well as the information that is required to recalculate it (i.e., academic performance in relevant exams and whether the applicant has a certified disadvantaged status).

The administrative data on students who applied during their senior year of high school is merged with the National Assessment of Basic Competencies (NABC) dataset based on demographic variables. The NABC measures numeracy and literacy skills in a standardized way. Since 2008 it has covered all students in the 6th, 8th, and 10th grades who attended school on the day of the exam (prior to 2008 it covered a sample). The NABC dataset includes self-reported survey measures of socioeconomic status (e.g., parental education, home possessions, etc.). Shorrer and Sóvágó (2017) construct an NABC-based socioeconomic-status (SES) index, following Horn (2013). This index consists of three subindices: an index of parental education, an index of home possessions (number of bedrooms, cars, books, computers, etc.), and

[24]We are abusing terminology here, since the formal definition of a contract includes the student's identity.

[25]The Hungarian Higher Education Application Database (FELVI) is owned by the Hungarian Education Bureau (Oktatasi Hivatal). The data were processed by the Hungarian Academy of Sciences Centre for Economic and Regional Studies (HAS-CERS).

[26]The dataset reports the first 6 contracts on an applicant's ROL as well as the applicant's allocation, in case it was ranked lower. It also specifies the number of contracts on each ROL. In 93.6% of the cases, we observe the complete ROL.

an index of parental labor market status.[27]

Shorrer and Sóvágó's dataset also includes microregion-level annual unemployment rates published by the National Employment Service in 2008, with a territorial breakdown consisting of 174 units. Finally, it includes the per capita gross annual income for all 3,164 localities for each year of the sample, calculated based on information published by the Hungarian Central Statistics Office.

4.3 Student rank-order lists

We now ask: *Is the data consistent with the assumptions of our model on the distribution of student ROLs?* Namely, are ROLs "short" (Roth and Peranson, 1999), and are applicants from lower socioeconomic backgrounds more likely to rank funded positions only, as our leading example suggests? The answer is positive.

We find that 93.6% of the ROLs rank up to six contracts, and 99.1% of the ROLs are shorter than 10 contracts. In Table 1 we compare the characteristics of applicants who submitted ROLs ranking funded contracts exclusively with those of applicants who submitted ROLs ranking both funded and unfunded contracts.[28] We find that, on average, applicants who ranked funded contracts exclusively come from a lower SES background, and have a stronger academic record.

The fact that applicants with stronger academic performance were more likely to submit an ROL ranking funded contracts exclusively can be explained by these applicants expecting to be able to gain admission to a funded contract. In case they are not admitted with funding, they may prefer to retake some exams and reapply the following year rather than pay tuition (Krishna, Lychagin and Robles, 2015), or they may be optimistic enough about their chances of admission to be (nearly) indifferent between their ROL and another ROL that ranks unfunded contracts (Chen and Pereyra Barreiro, 2015; Artemov, Che and He, 2017). Since SES is positively correlated with

[27] The objectives of the NABC are similar to those of the OECD Program for International Student Assessment (PISA). The NABC-based socioeconomic-status index resembles the PISA economic, social, and cultural status (ESCS) indicator.

[28] The third group, applicants who submitted ROLs ranking only unfunded contracts, is small and includes ineligible students (e.g., applicants from countries outside the European Economic Area) and applicants who probably made mistakes (Rees-Jones, 2017; Hassidim et al., 2017).

academic ability (Table 2), this pattern likely leads our analysis to understate the true scope for reallocating financial aid from wealthy applicants to needy ones.

Table 1: Characteristics of applicants who submitted ROLs with funded contracts only

Dependent variable	Funded contracts only						
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
NABC-based SES index	-0.062***	-0.069***					
	(0.0017)	(0.0017)					
11th-grade GPA (1-5)		0.077***		0.094***		0.094***	0.093***
		(0.0023)		(0.0012)		(0.0012)	(0.0012)
Income (1,000 USD)			-0.038***	-0.039***			
			(0.0006)	(0.0006)			
Unemployment (%)					0.008***	0.008***	
					(0.0002)	(0.0002)	
Observations	78064	78064	284701	284701	284701	284701	284701
R^2	0.017	0.032	0.016	0.038	0.007	0.028	0.021

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Notes: The regression coefficients are conditional on a year fixed effect and an indicator for missing values. Robust standard errors are in parentheses. The sample includes all ROLs, excluding those that ranked unfunded contracts only. In Columns 1 and 2 we restrict the sample to high-school-senior applicants, the population that was matched to the NABC data. The NABC-based SES index was matched to the main dataset based on 5 variables (year and month of birth, gender, school identifier, and four-digit postal code). It is normalized to have a mean of 0 and a standard deviation of 1 in the population of high-school students. Income stands for the per capita gross annual income in the locality where the applicant resided.

4.4 Stable allocations

We proceed to the main question of interest: *Is the set of stable allocations in the Hungarian college admissions market large, and can colleges successfully manipulate the mechanism?* As we have established above, the set of stable allocations in a college admissions market does not, generally, admit a lattice

Table 2: Academic ability and socioeconomic status

Dependent variable	Grade-11 GPA		
	(1)	(2)	(3)
NABC-based SES index	0.085***		
	(0.0028)		
Unemployment (%)		-0.005***	
		(0.0004)	
Income (1,000 USD)			0.016***
			(0.0011)
Observations	78133	284701	284701
R^2	0.325	0.056	0.056

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Notes: The regression coefficients are conditional on a year fixed effect and an indicator for missing values. Robust standard errors are in parentheses. The sample includes all ROLs, excluding those that ranked unfunded contracts only. In Column 1 we restrict the sample to high-school-senior applicants, the population that was matched to the NABC data. The NABC-based SES index was matched to the main dataset based on 5 variables (year and month of birth, gender, school identifier, and four-digit postal code). It is normalized to have a mean of 0 and a standard deviation of 1 in the population of high-school students. Income stands for the per capita gross annual income in the locality where the applicant resided.

structure with respect to same-side preferences. Thus, we are unable to characterize it fully using standard methods. Moreover, data limitations complicate our ability to verify stability.

Instead, we assume that the strategic unit on the colleges' side is a field of study – the unit that shares priorities and budget – and that its priorities, which are determined by the government, represent the field's true preferences.[29] We ask how much each field can improve its yield by applying market power over students who are placed in the field under DA when all colleges are truthful. Since each field of study offers multiple programs in multiple locations, we conservatively assume only that each field is indifferent to the identities of the recipients of state funding, while keeping students' placement fixed. Specifically, we do not take a stance on the field's preferences with respect to transferring students from one program to another program within the same field.

Based on our conversations with admissions officers in multiple programs that participate in the IPMM, we feel very comfortable about the assumption that colleges care about financial aid only insofar as it affects the composition of the incoming cohort. Furthermore, as we discuss below, the empirical exercise makes a weaker assumption, namely, the assumption that, holding the assignment of students to programs in the field fixed, a field prefers to be assigned more students who meet the minimal requirements as long as quotas are not full, and thus to increase the total amount of tuition collected (from students and from the government).

Given a field, f, and a year, t, we focus on identifying students who receive financial aid in f although the next-highest-ranked contract on their ROL whose priority score cutoff they pass is the unfunded contract with the same program (i.e., the same study track at the same institution). The set of such students, MP_f^t, is a set of students over whom f has market power; i.e., f can safely refuse their funding.

There are 10,056 students who belong to some MP_f^t in our sample period. They correspond to approximately 8% of the tuition waivers offered by the state in this period. Namely, when the financial aid offered in all other programs is held constant, about 8% of the waivers have no effect on the receiving students' choice of program.

[29] Prior to 2008, each program had a separate quota of state-funded seats, and some discretion in determining the weights in the priority-score formula. Presumably, this meant that a higher number of students were facing a funding-quality trade-off and, therefore, programs had greater scope for applying local market power.

We are interested in knowing whether the field can improve the incoming cohort by refusing funding to students in MP^t_f. We are especially interested in knowing whether such behavior increases the total number of students attending *some* college.

To this end, we define the set DB^t_f of students who stand to *directly benefit* if f applies market power. Given a program, f, and a year, t, DB^t_f is the set of up to $\left|\mathrm{MP}^t_f\right|$ highest-priority-score year-t applicants who were unassigned or assigned to a contract (not with f) that they ranked lower than the funded contract with a program in f that had free capacity.

Since we do not have data on exact capacities, we say that a program has free capacity if no student was rejected from the unfunded contract. We take a partial-equilibrium approach and assume that if f has free capacity in some program, it can apply market power over members of MP^t_f and use the freed-up funds to admit students in DB^t_f, so that the resulting allocation will be stable.

This approach is conservative in several ways. First, we do not consider improvements that the field can make by coordinating across its various programs. Second, we ignore programs in which the field can improve only the composition of the student body without affecting the size of the incoming cohort (i.e., the fields can accept better students by reallocating funding from price-insensitive students *and* reallocating capacity from lower-ranked unfunded students). Had each program (or college) acted as an independent strategic unit, this second source of gains would have been substantial, as a large fraction of applicants apply only to programs in one field.

We perform the analysis separately for each field in each year. We find that 9,463 students stand to benefit directly. Of these, 5,886 students are not placed in any college in practice (i.e., under DA). Table 3 compares the characteristics of the groups. We find that the students who benefit from moving to another stable allocation (i.e., members of some DB^t_f), on average, come from lower SES relative to those who stand to lose from such a change (i.e., members of some MP^t_f), that they are more likely to live in a village, and that they are less likely to live in the capital, Budapest. They are also more likely to be female and to have graduated from a vocational high school. Mechanically, winners also have lower academic achievements.

Table 3: Characteristics of applicants in MP and DB

	DB		MP	
	mean	sd	mean	sd
Disadvantaged (dummy)	0.09	0.288	0.03	0.173
Unemployment rate (%)	7.95	4.668	6.77	4.162
Gross annual per capita income (1000 USD)	6.03	1.451	6.59	1.540
11th-grade GPA	3.77	0.777	3.90	0.793
Female	0.58	0.493	0.50	0.500
Secondary grammar school	0.64	0.479	0.68	0.467
Vocational school	0.32	0.468	0.26	0.439
Capital	0.14	0.348	0.26	0.439
County capital	0.21	0.405	0.21	0.406
Town	0.34	0.474	0.29	0.452
Village	0.31	0.463	0.24	0.430
Programs in ROL	3.31	1.295	2.25	1.009
Contracts in ROL	3.70	1.773	4.21	2.273
Observations	9,463		10,056	
Unassigned under DA	5,886		0	

Notes: The table compares the characteristics of applicants over whom some field can apply market power (members of some MP^t_f) with those of students who stand to benefit directly from moving to another stable matching allocation (members of some DB^t_f). The sample covers the years between 2009 and 2011. Each year, approximately 60,000 students are assigned to college through DA, of which approximately 70% receive funding.

5 Discussion

We have shown that using student-proposing DA in a college admissions market is akin to allocating financial aid based on merit. This result generalizes to the college-proposing version of DA, the mechanism used in Turkish college admissions (Balinski and Sönmez, 1999).[30] A report by the World Bank states that the situation in this country "is akin to giving a large number of scholarships in each institution on the basis of merit." The report adds that "merit-based scholarships to be funded by government make sense if high caliber students need extra inducements for entering higher education. It is not obvious that Turkish students need such inducement," and that "an important group to target would be students from less privileged backgrounds, either in terms of income, regions, ethnicity or gender." Our findings provide support for these assertions and offer guidance on how to implement alternative policies.

We have established that the set of stable allocations in large college admissions markets is large, and that centralized two-sided college admissions markets that use the student-proposing version of DA leave much room for colleges to strategize. These findings stand in sharp contrast to the findings on markets without contracts. Our results suggest that colleges have an incentive to provide financial aid based on need rather than merit, even if they do not have preferences for equity or social justice and their only goal is to maximize the quality of their incoming cohort. This, in turn, implies that colleges have an incentive to collect information on other agents in the market (both students and colleges).

Since different stable allocations in college admissions markets differ substantially, market designers are facing economically meaningful trade-offs, even when concentrating on stable allocations. This observation suggests that answering "classic" questions in the theory of two-sided matching markets in the college admissions setting may be a fruitful direction. A natural question, for example, is: *How does one find the stable allocation that matches the most students?* Understanding the structure of the set of stable allocations in college admissions environments is, therefore, a promising research direction.

[30]In Turkey, private colleges are obligated by law to offer a full scholarship to, at least, 15% of their student body. Additionally, most institutions offer multiple contracts in the same program: a subsidized morning schedule, and a more expensive evening schedule (Akar, 2010; Hatakenaka, 2006).

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A Stability and the Core

In this appendix we show that in college admissions environments, the set of stable allocations coincides with the (weak-domination) core (Roth and Postlewaite, 1977), which is a subset of the (strict-domination) core.[31]

Proposition 3. *In college admissions environments, the set of stable allocations is equal to the weak-domination core.*

Proof. The proof closely follows Proposition 5.36 in Roth and Sotomayor (1990). Assume that the allocation Y is not stable. Then there exist a college, c, and $Z \subset X_c \setminus Y$ such that $Z_i \subset \mathrm{Ch}_i(Z_i \cup Y_i)$ for all $i \in S \cup C$. Thus, Y is weakly dominated by the coalition consisting of c and the collection of students involved in contracts in $\mathrm{Ch}_c(Z_c \cup Y_c)$, through the allocation $\mathrm{Ch}_c(Z_c \cup Y_c)$.

In the other direction, let Y be an allocation not in the core. If it is not individually rational, we are done. Otherwise, it is weakly-dominated by another allocation Y' via some coalition $A \subset S \cup C$. Hence, there is some college or student in A that strictly prefers Y' to Y. Since preferences are strict, there is at least one student $s \in A$ who gets a different allocation under Y', and hence strictly prefers Y' to Y and to the outside option. Since Y' is individually rational, Y'_s consists of a single contract (s, c, t) where $c \in A \cap C$. Since preferences are strict, c strictly prefers Y' to Y. Set $Z := \mathrm{Ch}_c(Y \cup Y')$. Then $Z_c \subset \mathrm{Ch}_c(Z_c \cup Y_c)$ by the irrelevance of rejected contracts (Aygün and Sönmez, 2013), since $Z = \mathrm{Ch}_c(Y \cup Y')$ and $Z_c \cup Y_c \subset Y \cup Y'$. Additionally, if i is a student involved in a contract in Z, then $Z_i \subset \mathrm{Ch}_i(Z_i \cup Y_i)$ because $i \in A$ and thus i prefers his allocation under Y' to his (individually rational and hence singleton) allocation under Y. Thus, Y is not stable. $\square$

B Proof of Theorem 2

We now prove the theorem for the general case. We require some additional notation.

[31] An allocation belongs to the (strict-domination) core if there does not exist a coalition of agents that can all get strictly higher utility by matching only among themselves. The definition of the weak-domination core allows some members of the coalition to get the same utility.

Given a college c in C^n, let $\hat{t}_c$ be the maximal $t \in T^n$ such that $0 < q^c_{\hat{t}_c}$. Similarly, let $\bar{t}_c$ be the maximal $t \in T^n$ such that $0 < q^c_{\bar{t}_c} < q^c_{\bar{t}_c - 1}$. The existence of $\hat{t}_c$ and $\bar{t}_c$ is assured by Condition 5 in the definition of a sequence of uniform random markets.

Given a college h in C^n, an ordered selection of $\bar{t}_h + 1$ students in S^n, $\left(p, r_1, r_2, ..., r_{q^h_{\bar{t}_h}}\right)$, and another college, c in C^n, let the event $E^n\left(p, r_1, r_2, \ldots, r_{q^h_{\bar{t}_h}}, h, c\right)$ denote the case where:

1. College h ranks lower-index r_j's higher on its master list, and ranks all r_j's higher than p. Formally, $r_j \gg_h r_i \gg_h p$ for all $1 \leq j < i \leq q^h_{\bar{t}_h}$.

2. The only students who find contracts with h acceptable are the members of $\left\{p, r_1, r_2, ..., r_{q^h_{\bar{t}_h}}\right\}$. Formally, for all $s \in S^n \setminus \{p, r_1, r_2, ..., r_{q^h_{\bar{t}_h}}\}$ and all $t \in T^n$, $\emptyset \succ_s (s, h, t)$.

3. The only student who finds contracts with c acceptable is p. Formally, for all $s \in S^n \setminus \{p\}$ and all $t \in T^n$, $\emptyset \succ_s (s, c, t)$.

4. All r_j's prefer to be placed in h under any terms to any contract with another college. Formally, for all $1 \leq j \leq q^h_{\bar{t}_h}$, and for all $z \in X_{r_j} \setminus X_h$, $(r_j, h, 0) \succ_{r_j} z$.

5. The most desirable contracts for p are $(p, h, |T| - 1), (p, h, |T| - 2), \ldots,$ $(p, h, \bar{t}_h)$, followed by $(p, c, |T| - 1), (p, c, |T| - 2), \ldots, (p, c, 0)$. Formally, for all $z \in X_p$, $z \succ_p (p, c, |T| - 1)$ if and only if $z \in \{(p, h, t) | t \geq \bar{t}_h\}$, and $z \succ_p (p, c, 0)$ if and only if $z \in \{(p, h, t) | t \geq \bar{t}_h\} \cup \{(p, c, t) | t > 0\}$.

Note that in the event $E^n(p, r_1, r_2, ..., r_{q^h_{\bar{t}_h}}, h, c)$, student p and colleges h and c have multiple stable allocations. The stable allocation resulting from DA includes $(r_1, h, \hat{t}_h)$ and $(p, c, \hat{t}_c)$. But another stable allocation involves a contract of the form (p, h, t) for $t \geq \bar{t}_h$ (and some r_j receiving a lower level of financial aid at h). Note that college h can successfully manipulate DA (e.g., by declaring that allocations under which r_1 receives financial aid are not acceptable), but clearly college c cannot do likewise. Furthermore, the two colleges have different numbers of students assigned to them in different stable allocations.

Lemma 2. *There exists $L > 0$ and n', such that for all $n > n'$ and for each college $h \in C^n$, the event*

$$E_h^n = \bigcup_{(p,r_1,r_2,...,r_{q^h_{\bar{t}_h}},c) \in S^n \times ... \times S^n \times C^n} E^n(p, r_1, r_2, ..., r_{q^h_{\bar{t}_h}}, h, c)$$

has a probability bounded below by L.

Proof. For sufficiently large n, given a selection of h, there are at least $\binom{\lfloor \frac{n}{\lambda} \rfloor}{q^h_{\bar{t}_h}+1}$ selections of $(p, r_1, r_2, ..., r_{q^h_{\bar{t}_h}})$ that meet the first condition. Given such a selection and a selection of one of the $n-1$ other colleges $c \neq h$, the probability that the other conditions are met is bounded below by

$$\underbrace{(1 - \frac{k}{n-1})^{\lceil 2\lambda n \rceil}} \times \underbrace{\frac{1}{n \cdot k^{|T|-1}}} \times \underbrace{\frac{1}{n^2 \cdot k^{2|T|}}},$$

where each term in this expression corresponds to an independent[32] requirement from the definition of the event $E^n(p, r_1, r_2, ..., r_{q^h_{\bar{t}_h}}, h, c)$. Moreover, since, given h, the events $E^n(p, r_1, r_2, ..., r_{q^h_{\bar{t}_h}}, h, c)$ are disjoint, the probability of their (disjoint) union is greater than

$$(n-1)\binom{\lfloor \frac{n}{\lambda} \rfloor}{q^h_{\bar{t}_h}+1} \times (1 - \frac{k}{n-1})^{\lceil 2\lambda n \rceil} \times \frac{1}{(n \cdot k^{|T|-1})^{q^h_{\bar{t}_h}}} \times \frac{1}{n^2 \cdot k^{2|T|}},$$

which converges to a positive constant (that depends on $q^h_{\bar{t}_h}$) as n grows large. Since $q^h_{\bar{t}_h} < \bar{q}$ by Condition 3 in the definition of uniform random markets, $q^h_{\bar{t}_h}$ can take only finitely many values; hence, taking the minimum of the limits and subtracting some small ε suffices.

□

Proof (of Theorem 2). The results on $\beta(n)$ and $\delta(n)$ follow directly from the lemma. Let $\chi_{E_h^n}$ denote the indicator of the event $\chi_{E_h^n}$. The results on $\gamma(n)$ follow from from the fact that the expectation of the random variable $\sum_{h \in C^n} \chi_{E_h^n}$ increases linearly in n, which implies that the expected number of colleges "playing the role of c" is large.[33] The result on $\alpha(n)$ follows from the

[32] To be precise, the second and third conditions are not independent. The first term of our bound applies to both of them simultaneously.

[33] This can be shown formally by changing the order of summation.

result on $\delta(n)$, and from the fact that each student has at most k colleges she prefers to the outside option. The result on $\eta(n)$ requires more work, and follows from Lemma 4 below. □

Lemma 3. *Given a profile of (complete) college master lists, at least $\frac{1}{3}$ of the students are ranked between $\frac{1}{4}|S|$ and $\frac{3}{4}|S|$ in at least $\frac{1}{4}$ of the lists.*

Proof. The fraction of students who appear in this half of the list of at least $\frac{1}{4}$ of the lists is equal to the probability that this condition is satisfied by a student drawn uniformly at random. Let $\chi_c(\cdot)$ denote the indicator variable that a student is in the top or bottom quarter of c's list. Then, by Markov's inequality,

$$\Pr\{\sum_{c\in C}\chi_c > \frac{3}{4}n\} \leq \frac{n/2}{3n/4} = \frac{4}{6},$$

which completes the proof. □

Denote by $M^n \subset S^n$ the collection of students who are ranked between $\frac{1}{4}|S^n|$ and $\frac{3}{4}|S^n|$ in at least $\frac{n}{4}$ of the lists. Consider a student in M^n and k colleges on whose master list the student is ranked between $\frac{1}{4}|S^n|$ and $\frac{3}{4}|S^n|$. Denote one of the colleges by $h \in C^n$, the student by $r_{q^h_{\bar{t}_h}} \in S^n$, and the other colleges by $\{c_i\}_{i=1}^{k-1} \in C^n$.

Select an additional college $c \in C^n$ and q^h_0 students in S^n, $\left(p, r_1, r_2, \ldots, r_{q^h_{\bar{t}_h}-1}, r_{q^h_{\bar{t}_h}+1}, \ldots, r_{q^h_0}\right)$. Finally, for each c_i select $\bar{q}$ different students whom the college ranks among the highest $\frac{1}{4}|S^n|$, $\{s^j_{c_i}\}_{j=1}^{\bar{q}}$. Let the event $E^n\left(p, r_1, r_2, ..., r_{q^h_0}, h, c, c_1, c_2, ..., c_{k-1}, \{\{s^j_{c_i}\}_{j=1}^{\bar{q}}\}_{i=1}^{k-1}\right)$ denote the case where:

1. College h ranks lower-index r_j's higher on its master list, and ranks p between $r_{q^h_{\bar{t}_h}}$ and $r_{q^h_{\bar{t}_h}+1}$. Formally, $r_j \gg_h r_i$ for all $1 \leq j < i \leq q^h_{\hat{t}_h}$ and $r_{q^h_{\bar{t}_h}} \gg_h p \gg_h r_{q^h_{\bar{t}_h}+1}$.

2. The only students who find contracts with h acceptable are the members of $\{p, r_1, r_2, ..., r_{q^h_0}\}$. Formally, for all $s \in S^n \setminus \{p, r_1, r_2, ..., r_{q^h_0}\}$ and for all $t \in T^n$, $\emptyset \succ_s (s, h, t)$.

3. The only student who finds contracts with c acceptable is p. Formally, for all $s \in S^n \setminus \{p\}$ and for all $t \in T^n$, $\emptyset \succ_s (s, c, t)$.

4. All r_j's prefer to be placed in h under any terms to any contract with another college. Formally, for all $1 \leq j \leq q_0^h$, and for all $z \in X_{r_j} \setminus X_h$, $(r_j, h, 0) \succ_{r_j} z$.

5. The most desirable contracts for p are $(p, h, |T|-1)$,$(p, h, |T|-2)$,...,$(p, h, \bar{t}_h)$, followed by $(p, c, |T|-1)$,$(p, c, |T|-2)$,...,$(p, c, 0)$. Formally, for all $z \in X_p \setminus \{(p, h, t)|t \geq \bar{t}_h\}$, $(p, c, |T|-1) \succ_p z$, and $z \succ_p (p, c, 0)$ if and only if $z \in \{(p, h, t)|t \geq \bar{t}_h\} \cup \{(p, c, t)|t > 0\}$.

6. The lowest-ranked r-student on h's master list, $r_{q_0^h}$, finds contracts only with $(h, c_1, c_2, ..., c_{k-1})$ acceptable, and prefers $(r_{q_0^h}, c_i, |T|-1)$ to $(r_{q_0^h}, c_j, |T|-1)$ if $i < j$. Formally, for all $u \in C^n$ and $t \in T^n$, $u \notin \{h, c_1, c_2, ..., c_{k-1}\} \implies \emptyset \succ_{r_{q_0^h}} (r_{q_0^h}, u, t)$, and $(r, c_i, |T|-1) \succ_{r_{q_0^h}} (r, c_j, |T|-1)$ if and only if $i < j$.

7. For each c_i the only students who find contracts with c_i acceptable are the members of $\{s_{c_i}^j\}_{j=1}^{\bar{q}}$. Formally, for all $i \in \{1, 2, ..., k-1\}$, for all $s \in S^n \setminus \{s_{c_i}^j\}_{j=1}^{\bar{q}}$, and for all $t \in T^n$, $\emptyset \succ_s (s, c_i, t)$.

8. For each c_i the members of $\{s_{c_i}^j\}_{j=1}^{\bar{q}}$ prefer to be placed in c_i under any terms to any contract with another college. Formally, for all $i \in \{1, 2, ..., k-1\}$, for all $1 \leq j \leq \bar{q}$, and for all $z \in X_{s_{c_i}^j} \setminus X_{c_i}$, $(s_{c_i}^j, c, 0) \succ_{s_{c_i}^j} z$.

Note that in the event $E^n\left(p, r_1, r_2, ..., r_{q_0^h}, h, c, c_1, c_2, ..., c_{k-1}, \{\{s_{c_i}^j\}_{j=1}^{\bar{q}}\}_{i=1}^{k-1}\right)$ the stable allocation that corresponds to the outcome of DA assigns students $\{r_1, r_2, ..., r_{q_0^h}\}$ to h, and student p to c. But in another stable allocation, college c and student $r_{q_0^h}$ receive no assignment, and students $\{p, r_1, r_2, ..., r_{q_0^h-1}\}$ are assigned to h, where one of the students in $\{r_1, r_2, ..., r_{q_{\bar{t}_h}^h}\}$ receives a lower level of financial aid relative to the stable allocation that corresponds to the outcome of DA.

Lemma 4. *There exists $L > 0$ and n', such that for $n > n'$, the probability that the event $E^n\left(p, r_1, r_2, ..., r_{q_0^h}, h, c, c_1, c_2, ..., c_{k-1}, \{\{s_{c_i}^j\}_{j=1}^{\bar{q}}\}_{i=1}^{k-1}\right)$ occurs for some selection of arguments is greater than L.*

Proof. Conditional on the selection of $\left(p, r_1, r_2, ..., r_{q_0^h}, h, c, c_1, c_2, ..., c_{k-1}, \{\{s_{c_i}^j\}_{j=1}^{\bar{q}}\}_{i=1}^{k-1}\right)$

being valid,[34] the probability of the event is bounded below by

$$(1-\frac{k}{n-2k})^{2\lambda n}\times(\frac{1}{k})^{|T|\bar{q}}\cdot(\frac{1}{n})^{q_0^h}\times(\frac{1}{k})^{2|T|}\cdot(\frac{1}{n})^{2}\times(\frac{1}{n})^{k-1}\times(1-\frac{k}{n-2k})^{k\lambda n}\cdot(\frac{1}{n})^{\bar{q}(k-1)}\times(\frac{1}{k})^{k|T|}.$$

This expression behaves asymptotically like

$$\bar{C}\cdot(\frac{1}{n})^{q_0^h}\cdot(\frac{1}{n})^{k+1}\cdot(\frac{1}{n})^{\bar{q}(k-1)}=\bar{C}\cdot(\frac{1}{n})^{q_0^h+\bar{q}(k-1)+k+1}$$

for some positive $\bar{C}$.

We next use Lemma 3 repeatedly and note that there are at least $\frac{1}{3}|S^n|$ valid ways to choose p. Moreover, given such a selection, there are at least $\binom{\lfloor\frac{n}{4}\rfloor}{k}$ ways to choose h and $\{c_i\}_{i=1}^{k-1}$. And, given such a selection, there are at least $\binom{\lfloor|S^n|/4\rfloor}{q_0^h+(k-1)\bar{q}}$ valid selections of $(r_1,r_2,...,r_{q_0^h})$ and $\{\{s_{c_i}^j\}_{j=1}^{\bar{q}}\}_{i=1}^{k-1}$.

To complete the proof, we note that for different selections of

$$\left(p,r_1,r_2,...,r_{q_0^h},h,c,c_1,c_2,...,c_{k-1},\{\{s_{c_i}^j\}_{j=1}^{\bar{q}}\}_{i=1}^{k-1}\right)$$

the events $E^n\left(p,r_1,r_2,...,r_{q_0^h},h,c,c_1,c_2,...,c_{k-1},\{\{s_{c_i}^j\}_{j=1}^{\bar{q}}\}_{i=1}^{k-1}\right)$ are disjoint. Thus, the expected number of students who are matched in some stable allocation, but are unmatched in another one, which is greater than the expected number of events $E^n\left(p,r_1,r_2,...,r_{q_0^h},h,c,c_1,c_2,...,c_{k-1},\{\{s_{c_i}^j\}_{j=1}^{\bar{q}}\}_{i=1}^{k-1}\right)$ that are realized, is bounded below by $\hat{C}n$ for some positive $\hat{C}$.

□

C Weaker Distributional Assumptions

In this appendix, we formalize the claim that Theorem 2 generalizes to a broad class of distributions over students' preferences.

Proposition 4. *Let $\{\tilde{\Gamma}^n\}_{n=1}^{\infty}$ be a sequence of uniform random markets, and let $\{\tilde{\Gamma'}^n\}_{n=1}^{\infty}$ be another sequence of random markets that differs only in the*

[34] We use the term "valid" when we refer to conditions on colleges' preferences, to emphasize that these are arbitrary (i.e., not assumed random).

distribution of students' preferences.[35] *Then, Theorem 2 holds for* $\{\tilde{\Gamma'}^n\}_{n=1}^{\infty}$ *so long as there exists some* $\bar{C} > 0$ *and* $\alpha \in (0,1)$ *such that for sufficiently large* n*, at least a fraction* α *of the events*

$$E^n(p, r_1, r_2, ..., r_{q^h_{\tilde{t}_h}}, h, c)$$

and

$$E^n\left(p, r_1, r_2, ..., r_{q^h_0}, h, c, c_1, c_2, ..., c_{k-1}, \{\{s^j_{c_i}\}_{j=1}^{\bar{q}}\}_{i=1}^{k-1}\right)$$

with valid selections of colleges are at least $\bar{C}$ *times as likely in* Γ'^n *as they are in* Γ^n*.*

Proof. Follows immediately from our proof of Theorem 2. □

Allowing the preference-generating process from the body of the paper to draw uniformly from all permutations (not just acceptable ones) satisfies the conditions of the proposition, as does allowing k, the number of colleges acceptable to each student, to be student-specific. The conditions also hold in sufficiently thick regular sequences of random markets (defined below), a broad family of sequences that generalizes the structure studied in Kojima and Pathak (2009) to the college admissions setting.

Let a *random market* be a tuple $\tilde{\Gamma} = \left\langle S, C, T, \{\succ_c, \gg_c, q^c_t\}_{t\in T}^{c\in C}, k, \kappa, \mathcal{D}\right\rangle$, where k is an integer greater than one, κ is a number in $(0,1)$, $\succ_c$ represents college c's strict preferences (which must be consistent with its master list, $\gg_c$, and its list of quotas, $\{q^c_t\}_{t\in T}$), and $\mathcal{D} := \{p_c\}_{c\in C}$ is a distribution on C. A random market induces a college admissions market by drawing, for each student independently at random, preferences in the following way.[36]

- Step 0: Set $t = 1$, $A = \emptyset$, $B = \emptyset$, and let R be an empty ROL.
- Step $t \leq k|T|$: With probability κ proceed to step t.1. Otherwise, proceed to step t.2.
- Step t.1: If there are contracts with k different colleges on R, stop and set R as the student's preferences, where other contracts are not

[35]Below, we present a definition of a random market that makes restrictions on the distribution of students' preferences. With a slight abuse of notation, we use the same term but do not make any restrictions.

[36]Each randomization in the algorithm is independent.

acceptable.[37] Otherwise draw a college, c, according to $\mathcal{D}$. If c is in $A \cup B$, repeat. Otherwise, append the most generous contract with c to the ROL , add c to B, and continue to step $t+1$.

- Step $t.2$: If B is empty, continue to step $t.1$. Draw uniformly at random a college from B, $c \in B$. Append the most generous contract with this college that does not appear on the ROL R to R. If the terms of this contract are the lowest in T, remove c from B and add it to A. Continue to step $t+1$.

A sequence of random markets, denoted by $\{\tilde{\Gamma}^n\}_{n=1}^{\infty}$, is *regular* if there exist integers $k, l, \bar{q}$, and λ, all greater than one, and $\kappa \in (0,1)$ such that:

1. $|C^n| = n$ for all n,
2. $k^n = k$, $\kappa^n = \kappa$, and $T^n = \{0, 1, ..., l-1\}$ for all n,
3. $q_0^c \leq \bar{q}$ for all $c \in C^n$ and all n,
4. for all n, $c \in C^n$, and $s \in S^n$, $s \gg_c \emptyset$,
5. for all n and $c \in C^n$, there exist $t, t' \in T^n$ such that $q_t^c > q_{t'}^c > 0$, and
6. $\frac{1}{\lambda} n \leq |S^n| \leq \lambda n$, for all n.

A regular sequence of random markets, $\{\tilde{\Gamma}^n\}_{n=1}^{\infty}$, is *sufficiently thick* if there exist $\rho > 0$, $\omega \in (0,1)$, and an integer n' such that for all $n > n'$,

$$\frac{\max\limits_{c \in C^n} p_c^n}{\max\limits_{c \in C^n}{}_{\lceil \omega n \rceil} p_c^n} < \rho,$$

where $\max_i$ is the i-th highest element in a set. This condition means that the most popular college is at most ρ times as popular as the $\omega \times 100^{\text{th}} -$ tile college (i.e., the ratio of popularities does not grow without bound).

[37] Alternatively, proceed to step $t.2$, and stop when R is longer than $k|T|$.

D Weaker Assumptions on Colleges' Preferences

In this appendix, we relax the assumption that colleges care lexicographically more about the composition of the incoming cohort than they do about how financial aid is allocated among these students.

Definition 1. For each college, $c \in C$, and student $s \in S$, denote the percentile rank of student s on c's master list by $P_s^c := \frac{|\{s' | s' \gg_c s\}|}{|S|}$.

The feasible part of the choice functions described in the body of the paper can be represented as the argmax of the following utility function:

$$u_c(Z) = \begin{cases} -\infty & \text{if } Z_c \text{ violates one of } c\text{'s quotas} \\ \\ \sum\limits_{(s,c,t)\in Z_c} (1 - P_s^c + \epsilon(s,c,t)) & else \end{cases}$$

where $\epsilon(s,c,t)$ are sufficiently small and assure that there are no preference ties. That we do not require feasibility makes transparent the fact that the choice function satisfies the hidden-substitutes condition of Hatfield and Kominers (2015), as well as the other conditions of their Theorems 1–3: the law of aggregate demand and the irrelevance-of-rejected-contracts condition.

Next, we consider choice functions induced by a broader class of utilities:

$$u_c(Z) = \begin{cases} -\infty & \text{if } Z_c \text{ violates one of } c\text{'s quotas} \\ \\ \sum\limits_{(s,c,t)\in Z_c} (1 - P_s^c + \kappa(s,c,t)) & else \end{cases}$$

where $\kappa(\cdot,\cdot,\cdot) \geq 0$ is some arbitrary function.

We note that the above-mentioned conditions of Hatfield and Kominers (2015) still hold. Thus, DA terminates in a stable allocation.

Recall that the definition of a regular sequence of markets made no restriction on colleges' preferences beyond the assumptions on quotas and that all students are acceptable. The following theorem uses the same definition, but allows for colleges' choice functions that are represented by the above form, as long as the very top students are preferred to the lowest-ranked students, regardless of the contractual terms.

Theorem 3. *Given a regular sequence of uniform random markets, if* $\kappa(s,c,t) < x < 1$ *for all* n *and* $(s,c,t) \in S^n \times C^n \times T^n$*, then there exists* $\Delta > 0$ *such that:*

1. $\liminf_{n\to\infty} \alpha(n)/n > \Delta$*,*
2. $\liminf_{n\to\infty} \beta(n)/n > \Delta$*,*
3. $\liminf_{n\to\infty} \gamma(n)/n > \Delta$*, and*
4. $\liminf_{n\to\infty} \delta(n)/n > \Delta$*. Furthermore, if* $x < \frac{1}{2}$
5. $\liminf_{n\to\infty} \eta(n)/n > \Delta$*.*

Proof. That colleges' choice functions are such that DA terminates in a stable allocation for any profile of students' preferences follows from Hatfield and Kominers (2015). Thus, the set of stable allocations is not empty.

Consider a college, h, and $q^h_{\bar{t}_h}+1$ students, $s_1, s_2, ...$, all ranked among the highest $(1-x)|S^n|$ students on h's master list. Endow each of these students, s_i, with artificial preferences of the form $(s_i, h, l-1) \succ_{s_i} (s_i, h, l-2) \succ_{s_i} ... \succ_{s_i} (s_i, h, \bar{t}_h) \succ_{s_i} \emptyset$. Run DA on a market consisting of h, with h's true preferences, and students $\{s_i\}_{i=1}^{q^h_{\bar{t}_h}+1}$ with the artificial preferences. Our assumptions assure that one (and only one) of the students will remain unassigned. We denote this student by p and the others by $r_1, r_2, ..., r_{q^h_{\bar{t}_h}}$ consistently with their order on the master list, and consider the event $\bar{E}^n(p, r_1, r_2, ..., r_{q^h_{\bar{t}_h}}, h, c)$, which is defined as $E^n(p, r_1, r_2, ..., r_{q^h_{\bar{t}_h}}, h, c)$ was previously defined, except that we omit the restriction that p is ranked on h's master list lower than any r_i.

Our selection of p assures that in the event $\bar{E}^n(p, r_1, r_2, ..., r_{q^h_{\bar{t}_h}}, h, c)$, p is not assigned to h in the allocation resulting from DA. Furthermore, similarly to the proof of Theorem 2, there is another stable allocation where p is assigned to h and one of the r-students receives a lower level of financial aid. Moreover, h can manipulate DA and achieve this allocation. The key is that since p is in the top $(1-x)$ percent of h's master list, the gain from recruiting him, which is greater than x, more than compensates for any possible loss due to the change in allocation of financial aid, which is bounded above by x.

Finally, our selection of p was independent of students' preferences, and therefor students' preferences are independent of the labels (which depend only on colleges' preferences). Thus, the lower bound on the probabilities of valid events of the form $E^n(p, r_1, r_2, ..., r_{q^h_{\bar{t}_h}}, h, c)$ extends to events of the form $\bar{E}^n(p, r_1, r_2, ..., r_{q^h_{\bar{t}_h}}, h, c)$. Furthermore, there are still "many" ways to select $p, r_1, r_2, ..., r_{q^h_{\bar{t}_h}}$: the number shrinks by a factor of $(1-x)^{q^h_{\bar{t}_h}+1}$ but remains of the same order of magnitude.

This completes the proof of the first four parts of the theorem. For brevity, we do not provide here the proof of the fifth part, but the generalization of the argument above to the proof of Theorem 2 is analogous. The requirement that x be lower than one half stems from the fact that two students assigned to h change their assignment: one receives a lower level of aid still in h and the other one becomes unmatched. Thus, h may lose up to $2x$.

□

E Market Power with Unattainable Desirable Alternatives

We extend Example 3 to show that a college may possess market power over a student even if the student does not rank the contracts with the college contiguously.

Example 4. We add to Example 3 another (elite) college e, and a (genius) student g. The elite college has one seat and one level of funding ($q^e_0 = 1$, $q^e_1 = 0$), and its preferences are summarized by

$$(g, e, 0) \succ_e (r, e, 0) \succ_e (p, e, 0) \succ_e \emptyset.$$

Student g's first-choice college is e. Her preferences are given by

$$(g, e, 0) \succ_g (g, h, 1) \succ_g (g, h, 0) \succ_g (g, c, 0) \succ_g \emptyset.$$

Other students' preferences are now

$$(r, h, 1) \succ_r (r, e, 0) \succ_r (r, h, 0) \succ_r (r, c, 1) \succ_r (r, c, 0) \succ_r \emptyset,$$

and

$$(p, h, 1) \succ_p (p, c, 1) \succ_p (p, c, 0) \succ_p (p, h, 0) \succ_p (p, e, 0) \succ_p \emptyset.$$

Thus, the elite college and the genius applicant are the first choice of one another, and hence must be matched in any stable allocation. There are two stable allocations under these preferences: $\{(r, h, 1), (p, c, 0), (g, e, 0)\}$, which is the result of the student-proposing DA algorithm, and $\{(r, h, 0), (p, h, 1), (g, e, 0)\}$.

Had the elite college been interested in the rich applicant (e.g., if its capacity was 2), then the unique stable allocation would be $\{(r, h, 1), (p, c, 0), (g, e, 0)\}$. Intuitively, the fact that the rich applicant prefers the elite institution to the unfunded position at h, combined with the institution's willingness to accept the rich applicant, eliminates h's ability to apply market power over the rich applicant. That h can apply market power over r is a result of e not being interested in r, which means that r does not have an outside option he prefers.

F Evidence from the IPMM

We complement the analysis from Section 4 by studying data from the Israeli Psychology Master's Match. We provide evidence in support of our assumptions on the demand structure, and corroborate the predictions of our model.

Background

In Israel, admissions to graduate programs in psychology are highly selective. Each year, about 1,400 students graduate from a bachelor's program, but this does not certify them to serve as therapists. In order to become a therapist, one needs to complete a clinical graduate degree and later an apprenticeship. But seats in clinical programs are scarce: only 300 students are accepted each year and about 300 students are accepted to other, non-clinical, programs.

While the number of applicants – approximately 1,000 a year – far exceeds the number of available seats, departments of psychology still compete for top talent. In an attempt to attract "star" applicants, several departments offer a limited number of prestigious scholarships to selected students. In 2014, when this market was centralized, it was critical to allow these departments to continue to pursue this recruiting strategy. Thus, the version of DA that is used in this market allows programs to offer contracts with multiple funding levels, and allows applicants to rank these alternatives separately.[38]

[38] The mechanism used by the Israeli Psychology Master's Match is strategy-proof for

Data

We use the dataset prepared by Hassidim, Romm and Shorrer (2016). The data includes administrative match data, including ROLs and program reports from the 2014 and 2015 matches. Additionally, it includes the computer code for the market-clearing algorithm.

In 2014, 10 programs in 3 departments offered admission under multiple funding levels. The number increased to 15 programs in 4 departments in 2015. Funding levels ranged from approximately \$2,000 a year to approximately \$20,000 a year. The number of available scholarships was 25 in 2014, and 36 in 2015.

Each year about 1,000 students participated in the match. The number of ROLs ranking some contract with a program offering multiple funding levels was 271 in 2014, and grew to 458 in 2015, as a result of the growth in the number of programs offering admission with multiple levels of funding (i.e., if we ignore observations attributed to new programs only, the number remained almost constant).

Student rank-order lists

Participants ranked, on average, 4.32 contracts ($\sigma = 4.14$). About 37.2% of the participants ranked at least one of the programs that offered admission under multiple financial-aid levels. Of these, only 3.4% ranked only the funded contract in some program, but not the unfunded contract. Among the applicants who ranked both a funded and an unfunded contract, more than 90% ranked the funded contract first.[39] Among these applicants, the mean number of contracts ranked between a funded contract and the unfunded contract with the same program was 0.34. In 82.6% of the cases, the number was zero.

Stable allocations

While we have access to all match data, there are still data limitations that do not allow us to calculate the set of stable allocations fully. The main issue

students. For more details about the market and the mechanism, see Hassidim, Romm and Shorrer (2017).

[39]We believe that other applicants made a mistake. For details, see Hassidim, Romm and Shorrer (2016).

is that only the parts of the departments' preferences that were required to calculate the outcome of DA were elicited. And departments, which typically offer several study programs, often have complex preferences. An additional issue is that we are not aware of an efficient way to calculate the set of stable allocations.

Instead, we take an approach similar to the one we used in the Hungarian dataset for detecting applicants over whom the program may be able to apply market power (MP_c). Next, we declare them as ineligible for funding in that program (by changing the department's preference report) and rerun the match. Finally, we check if the match is stable with respect to true preferences (i.e., if the applicant and the department are part of a blocking coalition of the resulting match). To do so, we require the assumption that programs do not care directly about the identities of the recipients of financial aid, but only about the quality of the incoming cohort. Based on our discussions with department chairs and recruiting committees during the design of the centralized clearinghouse, we are very comfortable with this assumption.

We find that, with a few exceptions, programs have market power over the recipients of financial aid. And while they can reallocate funding among admitted students, applying market power will not improve the quality of their incoming cohort. Our findings are reminiscent of Claim 1 in Example 1, which is not surprising in light of the fact that in 82.6% of the cases the funded and unfunded contracts in the same program were ranked consecutively.[40]

[40] In the 2016 match, two out of the four departments that had offered multiple levels of funding decided to offer identical terms to all students admitted to the same program.